MECHANISM OF HEAT TRANSFER IN VACUUM SUPERINSULATION

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This paper shows that the experimentally observed point of inflection on the temperature curve in vacuum superinsulation is due to the nonuniform residual gas pressure in the layers and the significant contribution of the gas conductivity to the total heat transfer.

It has been shown experimentally [1, 2] that the curve of the temperature distribution in vacuum superinsulation can have a point of inflection. Figure 1 shows a typical temperature distribution in insulation consisting of aluminum foil with glass-paper spacers at boundary temperatures of 300–77°K and a pressure of $2 \cdot 10^{-5}$ N/m^2 surrounding the specimen. We will show that the point of inflection on the temperature curve must be due to the variable residual pressure of the gas in the layers, which makes a significant contribution to the total heat transfer [3].

At the point of inflection we must have

$$\frac{d^2T}{dx^2} = 0, \tag{1}$$

which can be put in the form

$$\frac{dT}{dx}\frac{d}{dT}\left(\frac{dT}{dx}\right) = 0. \tag{2}$$

It should be noted that the temperature distribution in the screens in the insulation is discrete. However, since there is a large number of screens (each of which we will regard as an isothermal surface) the temperature of adjacent screens differs insignificantly and, hence, we can regard the temperature distribution curve as continuous and differentiable.

The temperature gradient dT/dx can be found from the differential energy equation [4]. Calculations show that in the considered insulation the heat transfer due to mass transfer (i.e., the removal of heat with the pumped-out gas molecules) is not more than 1% in steady-state conditions. Hence, we can neglect the terms of the equation which give the heat transfer due to mass transfer. In this case the energy equation becomes the usual Fourier equation

$$\frac{dT}{dx} = -\frac{q}{\lambda(T,\ x)}. \tag{3}$$

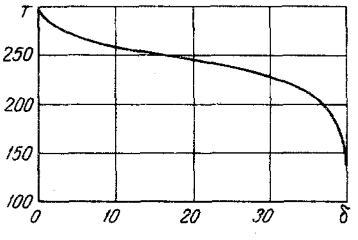

Fig. 1. Distribution of temperature over thickness of specimen of vacuum superinsulation at boundary temperatures 300–77°K (T, deg K: δ, mm).

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Since in the general case the properties of the considered insulation may depend on the depth coordinate in the insulation (owing to the variable residual gas pressure in the layers and nonuniform compression of the insulation, which may lead to a change in the contact conductivity), we will regard the thermal conductivity as a function of T and x, i.e., $\lambda = \lambda(T, x)$.

Heat transfer between adjacent screens is effected by radiation, the thermal conductivity of the spacer material, and the conductivity of the residual gas, so that we can write

$$\lambda(T, x) = \lambda_{\rm rad}(T) + \lambda_{\rm s.\,s}(T, x) + \lambda_{\rm g.\,eff}(T, x). \tag{4}$$

We will evaluate each component of this relationship.

The transfer of heat by radiation between adjacent screens can be introduced in the following way:

$$\lambda_{\rm rad}(T) = \frac{q_{\rm rad}}{\Delta T_i}\frac{\delta}{N}. \tag{5}$$

Since the SBP-M glass spacer material causes hardly any reduction of radiative heat transfer [5], we can regard this material as "transparent." Then the radiative heat transfer between adjacent screens is

$$q_{\rm rad} = \sigma\varepsilon_{\rm red}(T)\,(T_i^4 - T_{i+1}^4) = \sigma\varepsilon_{\rm red}(T)\,[T_i^4 - (T_i - \Delta T_i)^4]. \tag{6}$$

Neglecting terms of the second and higher order in ΔT_i in relationship (6), and using (5), we obtain

$$\lambda_{\rm rad}(T) = 4\sigma\varepsilon_{\rm red}(T)\,T^3\frac{\delta}{N}, \tag{7}$$

where we replace the discrete values of the screen temperatures T_i by a continuous function T(x) in view of the assumption of continuity of the temperature curve, and

$$\varepsilon_{\rm red}(T) = \frac{\varepsilon(T)}{2 - \varepsilon(T)} \tag{8}$$

if the emissivity of adjacent screens is assumed approximately equal in view of the small difference in their temperatures and the weak dependence of ε on the temperature [5].

We will assess the role of different factors in the transfer of heat through the solid substance (spacer). The resistance $R_{\rm s.s}$ to heat transfer in the solid substance between the screens is composed of the resistances of the contacts R_c and spacer material R_m and, hence, can be written as

$$R_{\rm s.\,s} = R_{\rm c} + R_{\rm m} \tag{9}$$

where

$$R_{\rm c} \sim 1/\lambda_{\rm c} \quad \text{and} \quad R_{\rm m} \sim 1/\lambda_{\rm m}.$$

Since the condition $\lambda_m \gg \lambda_c$ is fulfilled, we can assume $R_{\rm s.s} \sim 1/\lambda_c$, and $\lambda_{\rm s.s} = \lambda_c$. Hence, the main role in heat transfer through the solid substance is due to contact conductivity and relationship (4) takes the form

$$\lambda(T, x) = \lambda_{\rm rad}(T) + \lambda_{\rm c}(T, x) + \lambda_{\rm g.\,eff}(T, x). \tag{10}$$

Since $dT/dx \neq 0$ in the region of the point of inflection (see Fig. 1), then from expression (2) at the point of inflection we obtain the relationship

$$\frac{d}{dT}\left(\frac{dT}{dx}\right) = 0,$$

which in view of expression (3) can be written in the form

$$\frac{d\lambda(T, x)}{dT} = \frac{\partial\lambda(T, x)}{\partial T} + \frac{\partial\lambda(T, x)}{\partial x}\frac{dx}{dT} = 0. \tag{11}$$

Using expressions (7) and (10), we obtain from condition (11) the following relationship for the point of inflection:

$$\frac{\partial\lambda_{\rm c}(T, x)}{\partial T} + \frac{\partial\lambda_{\rm c}(T, x)}{\partial x}\Big/\frac{dT}{dx} + \frac{12\varepsilon(T)}{2 - \varepsilon(T)}\sigma T^2\frac{\delta}{N} +$$

$$+\frac{8\sigma T^3\frac{\delta}{N}\frac{\partial\varepsilon(T)}{\partial T}}{[2-\varepsilon(T)]^2}+\frac{\partial\lambda_{g.eff}(T,x)}{\partial T}+\frac{\partial\lambda_{g.eff}(T,x)}{\partial x}\Big/\frac{dT}{dx}=0. \tag{12}$$

As we know [5], for aluminum foil $\partial\varepsilon(T)/\partial T > 0$. In addition, $0 < \varepsilon(T) < 1$. Hence, at $T > 0$ the condition

$$\frac{12\varepsilon(T)}{2-\varepsilon(T)}\sigma T_{in}^2\frac{\delta}{N}+\frac{8\sigma T_{in}^3\frac{\delta}{N}\frac{\partial\varepsilon(T)}{\partial T}}{[2-\varepsilon(T)]^2}>0 \tag{13}$$

is satisfied. The contact conductivity decreases with reduction of temperature and specific pressure on the contacts [6] and, hence, $\partial\lambda_c(T, x)/\partial T > 0$. Figure 1 shows the curve for the case where the mechanical load due to the weight of the insulation itself under the particular experimental conditions [2, 3] decreases with approach to the cold wall. In view of this we can write $\partial\lambda_c(T, x)/\partial x < 0$. In view of the condition $dT/dx < 0$, which follows from the choice of direction of the coordinates (see Fig. 1) and the monotonicity of the function $T(x)$ we finally obtain

$$\frac{\partial\lambda_c(T,x)}{\partial T}>0 \quad\text{and}\quad \frac{\partial\lambda_c(T,x)}{\partial x}\Big/\frac{dT}{dx}>0.$$

Hence, from relationship (12) we obtain the following condition at the point of inflection:

$$\frac{\partial\lambda_{g.eff}(T,x)}{\partial x}\Big/\frac{dT}{dx}+\frac{\partial\lambda_{g.eff}(T,x)}{\partial T}<0. \tag{14}$$

Thus, only the presence of appreciable and variable heat transfer through the gas can cause the appearance of a point of inflection on the temperature curve.

We evaluate the residual gas pressure at the point of inflection. From the kinetic theory of gases [8], on condition that the Knudsen number $Kn \gg 1$ (which is the case in the considered insulation [3]), we can write for the heat transfer by the gas between adjacent screens

$$q_g=\frac{1}{2}\,\frac{a}{2-a}\,\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi MT_i}\right)^{\frac{1}{2}}\Delta T_i p(x).$$

This relationship is applicable in our case, since owing to the great porosity of the filling material ($m > 0.9$) the gas molecules pass through it freely in a transverse direction and are stopped only by the screen surfaces. Introducing $\lambda_{g.eff}$ in the form $\lambda_{g.eff} = q_g\delta/N/\Delta T_i$, we obtain

$$\lambda_{g,eff}=\frac{1}{2}\,\frac{a}{2-a}\,\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi MT}\right)^{\frac{1}{2}}p(x)\,\frac{\delta}{N}. \tag{15}$$

Using relationship (15), from expression (14) for the point of inflection we can write

$$\frac{\partial\lambda_{g.eff}}{\partial x}\Big/\frac{dT}{dx}+\frac{\partial\lambda_{g.eff}}{\partial T}=\frac{1}{2}\,\frac{a}{2-a}\,\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi M}\right)^{\frac{1}{2}}\left(\frac{p(x)}{2T^{\frac{3}{2}}}+\frac{\frac{dp(x)}{dx}}{T^{\frac{1}{2}}\frac{dT}{dx}}\right)<0. \tag{16}$$

This expression, in view of the conditions $p(x) > 0$, $T(x) > 0$, and $dT/dx < 0$, has the following form:

$$\frac{p(x_{in})}{2T_{in}}-\frac{\left.\frac{dp(x)}{dx}\right|_{x=x_{in}}}{\left.\frac{dT}{dx}\right|_{x=x_{in}}}>0. \tag{17}$$

It follows from the obtained expression that in the layers of insulation in the region of the point of inflection the residual gas pressure is not zero. We can determine the numerical value of (17) by using relationships (12), (15), and (16) and conditions $\partial\lambda_c(T, x)/\partial T > 0$, $\partial\varepsilon(T)/\partial T > 0$, and $\partial\lambda_c(T, x)/\partial x < 0$:

$$\frac{p(x_{in})}{2T_{in}} - \frac{\left.\frac{dp(x)}{dx}\right|_{x=x_{in}}}{\left.\frac{dT}{dx}\right|_{x=x_{in}}} > \frac{24\varepsilon(T)\sigma T_{in}^{\frac{5}{2}}}{\frac{a}{a-2}\frac{\gamma+1}{\gamma-1}\left(\frac{R}{2\pi M}\right)^{\frac{1}{2}}} \tag{18}$$

Since the point of inflection is at a fairly high temperature we can state that in the region of the point of inflection there is either a considerable change in the residual gas pressure or the absolute value of the pressure is fairly appreciable.

Thus, the presence of a point of inflection on the temperature curve cannot be attributed to radiative heat transfer through the solid substance. The only way of obtaining a consistent explanation is to assume appreciable heat transfer through the residual gas in the layers of the insulation [3].

It is of interest to evaluate the order of magnitude of the residual gas pressure at the point of inflection. In the case where $dp(x)/dx > 0$ and is sufficiently large in absolute value the pressure at the point of inflection, as relationship (18) shows, may not be very high. In this case, however, we can expect fairly high residual gas pressures after the point of inflection. If at the point of inflection $dp(x)/dx < 0$ or is fairly low, we find from relationship (18) that

$$p(x_{in}) > 0.7 \text{ N/m}^2.$$

The last value is obtained on the assumption that the residual gas is air, and the numerical values in relationship (18) are: $a = 0.8$; $\varepsilon = 0.025$; $\gamma = 1.42$; $T_{in} = 260°K$.

The obtained value of the pressure agrees well with the postulated residual gas pressure distribution, given in [3], in the insulation layers. Hence, near the point of inflection we can expect an insignificant change in gas pressure, i.e., the point of inflection will lie close to the maximum of the curve of $p(x)$, which also corresponds with the data of [3].

Relationship (18) shows that the pressure at the point of inflection can be determined from the temperature at this point. The absence of a point of inflection on the temperature curve will indicate that the heat transfer due to the conductivity of the residual gas in the insulation is a small fraction of the total heat transfer.

It should be noted in conclusion that recent direct measurements of the pressure in insulation layers [7] confirm the presence of a residual gas pressure of up to 1.3 N/m^2 when the ambient pressure is $2.6 \cdot 10^{-3}$ N/m^2.

NOTATION

T is the temperature;
x is the coordinate of depth in the insulation;
$\lambda(T, x)$ is the local thermal conductivity within the insulation, including all kinds of heat transfer;
$\lambda_{rad}(T)$, $\lambda_{s.s}(T, x)$, and $\lambda_{g.eff}(T, x)$ are the conductivities due to radiation, solid substance, and gas between adjacent screens;
$q = const$ is the total specific heat flux through the insulation;
$q_{rad}(T)$ is the local specific heat flux due to radiation;
δ is the insulation thickness;
N is the number of metal screens in the insulation;
T_i and T_{i+1} are the temperatures of two adjacent screens;
$\Delta T_i = T_i - T_{i+1}$ is the difference in the temperatures of two adjacent screens;
ε_{red} is the reduced emissivity of the system of two parallel screens;
σ is the Stefan–Boltzmann constant;
$\varepsilon(T)$ is the emissivity of the screens;
$\lambda_m(T)$ is the local heat transfer coefficient of spacer material;
$\lambda_c(T, x)$ is the contact conductivity of spacer material;
$p(x)$ is the residual gas pressure in the insulation layers;
T_{in} and x_{in} are the temperature and coordinate of the point of inflection;
a is the accommodation coefficient of the screen material for the gas;

R is the universal gas constant;
M is the molecular weight of the residual gas;
$\gamma = c_p/c_v$ is the ratio of the specific heats at constant pressure and constant volume.

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